RANDOMNESS, COMPUTABILITY, AND EMBODIMENT

Abstract

Physical and mathematical concepts ultimately reflect the nature of the embodied human organism as well as its world. Non-computability in mathematics and randomness in physics refer to limits within an epistemic relationship between subject and object. However, non-computability signifies the ability of a self-reflective agent to transcend its own formulations. In contrast, natural randomness signifies the world's ability to transcend formalization in thought.

1. Subject-object

What does the physically embodied nature of an epistemic subject imply for the properties and uses of mathematics? In whole or in part, mathematics will be viewed here as a *simulation* created by the human organism for adaptive purposes, just as ordinary perception is a virtual reality that simulates the world for such reasons. The purpose of the mathematics simulation is to characterize the most general properties of the physical world in a powerfully abstract and compressed way, especially to facilitate prediction. The expressive possibilities within a simulation are set by how it is configured. Like any simulation, mathematics constitutes a stand-alone "toy" world that mirrors reality yet potentially diverges from it.

An epistemic subject is an embodied agent whose natural focus is its environment. If the agent is aware of its agency, it is capable of self-reference and has a dual focus: the world as such, but also its own experience and thought as such, which may include linguistic objects. Distinguishing the focus is often problematic. Science is a strategy to disentangle the subjective from the objective aspects of experience, first by restricting its language to external objects, whether tangible or abstract, thereby avoiding self-reference. (Hence, 'electron' is a scientific object while 'concept of electron' is not.) Then: further by eliminating idiosyncratic elements, imposing strict protocols, and standardizing interchangeable observers.[i] What escapes this net, however, is what standardized observers have in common that is not on the table for discussion—for example, properties belonging commonly to all observers at the species level. As a universal cognitive tool, mathematics confidently substitutes reason to represent collective past experience. Yet, mathematical objects conform to reason, and reason is (supposedly) universal, because they have alike been abstracted from the most generic and indisputable aspects of the world as seen by a particular organism.

Furthermore, the world is apparently subdivided into distinct objects with interrelationships. These implicitly involve the observer (forming a literal triangle in the case of two objects observed in space). The agent is motivated to categorize these relationships in various ways (cause/effect, closer/farther, uniformly or non-uniformly changing, etc.) The notion of mathematical function expresses in a precise but abstract way how one object of thought relates to another, especially over time, since change is of vital significance for the organism. Where possible, mathematics goes helpfully beyond noting qualitative relationships to express exactly how one "variable" changes continuously with another. (That is, the idea of *variable* is itself an abstraction.) The trade-off for this gain is that *only* those patterns or relationships are considered which can be so formulated. Thus, velocity is precisely expressed as uniform change of location; acceleration as uniform change of velocity; change of acceleration as a further differential. But these are ideals of analysis to which most real phenomena may not conform. Special relationships are identified as physically significant that can be expressed with such mathematical tools (equations or differentials of *n*th degree); but the complete reality of any given phenomenon—and certainly the world as a whole—may not correspond in detail to any collection of such expressions. How should Brownian movement, for example, be expressed?

Agents may claim that these relationships exist "objectively," rather than as their assertion or special observation. This reifying tendency itself is a consequence of the natural focus of embodied agents upon the environment that holds over them the power of life and death. Thus, limits of scientific knowledge center on prediction, but often as though predictability were an objective property of the externals concerned rather than reflecting the observer's needs and state of knowledge. For example, the concept of *information* in physics tends to be objectified as a quantifiable property of systems themselves, even as a fundamental physical entity. However, what we take to be information is relative to our needs and goals; indeed, any agent's actions and perceptions are at most *co-determined* by the world external to it. Ignoring the agent's role gives a skewed, though useful, view of the world.

Concerns about the power and limits of mathematics have ultimately to do with prediction in the real world. Apart from their intrinsic fascination, we care about potential discoveries in the mathematics simulation because they might affect how we fare in reality. If mathematics reflects nature's intrinsic self-consistency, then it is unsurprising to find it consistent with parts of natural reality newly discovered or outside familiar domains. (The question is then why *nature* is self-consistent, if indeed it is.) On the other hand, this gives a certain urgency to the consistency of mathematics itself, for without it the usefulness of the simulation is compromised.

One is naturally inclined to view the world in idealized ways that lend themselves to effectively deciding the truth of propositions and predicting the course of events. Hence, the law of excluded middle and the historic focus on systems describable with simple linear equations, manually solvable. Idealizing these capacities themselves resulted in the Laplacian faith that anything past or present could be calculated, given a precise enough input. The subsequent realization that non-linear processes prevail in the real world followed upon Poincaré's study of the 3-body problem and the advent of digital computers, through which 'deterministic chaos' was rediscovered by Lorenz. This computational finding had the real-world consequence that inputs could not be measured accurately enough to allow effective prediction of many phenomena humanly important. Precise inputs can be *specified*, but the outputs—though reliably deduced—might not correspond to reality. In other words, the world is not deterministic in the way Laplace's generation had hoped. There was never any reason to suppose that it is, save the wishful thought that the world should be commensurate with reason, the future predictable.

2. Map/territory

The determinism that was once supposed to be an inherent property of *nature* is instead a defining property of *equations* and other conceptual systems, by virtue of their being *products of definition*. The *map* is deterministic, not the territory. Determinism is a property of the mathematical simulation, not of physical reality in either the classical or quantum realm. In other words, causal processes in the natural world are not a matter of logical implication as they are in mathematics. Moreover, being empirical, all scientific knowledge is essentially statistical. The difference between the classical and quantum domains is the brand of statistics, which depends on whether the agent can keep track of individual particles (objects).

The "unreasonable effectiveness" of the mathematics simulation for describing the world is but a special instance of the effectiveness of cognition in general to relate the domain of nature to our mental maps of it. Behind the utility of prediction tacitly lies the human project of the mastery of nature and the creation of a specifically human environment. This artificial environment is conceptual—as well as, and often before, it is physical. The conceptual map bears a symbolic relation to the territory that functions for specific purposes. If the map seems to be a likeness or copy, this illusion is the result of a strange loop whereby a self-aware agent realizes it can conceive the territory only in the terms of the map. One must distinguish even the familiar cognitive map we call phenomenal consciousness, and which we take for reality on a daily basis, from the inaccessible domain it maps—which Kant called the *noumenon*.

The mapmaker (the human brain) is in the position of a pilot flying by instrument or a submarine navigator charting the underwater world. Like these characters in their capsules, the brain is sealed within the skull, with access to the putative outside only through electro-chemical signals, of which it must learn to make sense. *Unlike* these characters, it has never had any other access to the external world. This constrains the nature of the map in a unique way: it is not a literal image but a device that facilitates successful navigation—where success means simply that the ship is not destroyed. The navigator does not survive because the map is true (which puts the cart before the horse). Rather the map is "true" because the navigator survives!

3. Decision

Decisions are made by agents with respect to needs. Although never forthcoming, perfect information is desirable for action. The amount of information a message contains is related to the number of yes/no decisions required to characterize it unequivocally. If the agent cannot decide some questions, the information remains ambiguous, its quantity indefinite. An artifact (such as a message, theory, or model) is potentially characterized by precisely definite information, but there is no guarantee that the reality it models can be so characterized. Indeed, there is little reason to suppose that the world is a message! It is the agent's prerogative (and perhaps mandate) to *force* decisions where a justification has not been legitimized; but this merely reflects the biological and psychological need for certainty.

Perfect certainty pertains only to deductive truths—that is, things *defined* to be so in the first place (axioms) and things (theorems) that can be deduced from them according to rules that are universally accepted. Mathematical or logical propositions (being clearly defined) have the advantage that the community of mathematicians is at least marginally more likely to agree that a theorem has been proven than the scientific community is likely to agree that a theory is true. The nature of proof in the two instances is different, even if both are intersubjective and debatable. In contrast to the scholasticism it superseded, the whole idea behind the Scientific Revolution is that theories of nature should be demonstrated empirically rather than deductively.

Scientific consensus nowadays depends on conventions for evaluating statistical error—as in deciding, for example, whether a high energy experiment confirms the existence of a particle. Nevertheless, deduction is still a perennial ideal in modern science: the hope for a theory of everything rests on the notion that all physical properties should ultimately be deduced from a few first principles. Many physical laws were in fact logically deduced before being confirmed empirically. Yet, such deductions are only possible because they deal with universal properties of things whose more specific and contingent properties (such as weight) cannot contradict them. Hence, Galileo's conclusion that the weight of objects does not affect their rate of fall came to him deductively in a thought experiment: by supposing the contrary, it was clear that objects could not fall at different rates without contradicting the very fact of being separate objects! Similarly, it is clear that forces cannot be instantaneously transmitted (action at a distance) without violating common-sense notions of space and time.

It might seem surprising that the physical world should appear consistent and logically organized ("rational") in the way that math is; but this is no coincidence, because math was modelled on the world and not the other way around. Moreover, in truth the parallel is only partial. The very strength of science is to reveal the apparent *in*consistencies in nature—for example, the illogical behavior of quantum objects. That discrepancy stems from the fact that our habitual logic itself generalizes—and then proceeds to institutionalize in its "laws"—the behavior of things in the familiar macroscopic realm. The notion of *a priori* truth, gleaned from such observed behavior, is not absolute but rather characterizes those levels of cognition that are common, if not universal for the species, and thus come to seem non-contingent and necessary.

<u>4. Objects</u>

Mathematics deals with grand generalities concerning discernable "things." Computability theory is thus concerned mainly with the natural numbers, which abstract the properties of "objectness." However, these generalities have been formalized as the rules, operations, and entities of a deductive system—that is, a free-standing product of definition categorically distinct from the world itself. The truths of logic and of deductive systems are true by definition, whether or not they have been discovered yet or formally derived yet from currently known truths, and whether or not they represent known physical realities. Mathematical truth and physical truth are distinct sets, which overlap to an indefinite degree. As in the real world, we assume that some relationships in the mathematical simulation remain to be found, however complete the existing formalization. While this assumption is intuitive (though counter to the ideal of perfect knowledge), it was deductively proven by Gödel. It was further established that there are infinitely more non-computable numbers than computable ones in the mathematics simulation, paralleling the preponderance of "deterministically" chaotic processes in nature.

It seems useful to distinguish between those relationships within the natural world that must follow from its general properties (as formalized in logic and mathematics) from those that cannot be so deduced. The notion of computability may afford this distinction. Non-computability presents a limit on the ability of the mathematics (or any) simulation to capture the real world. While math is the most general and fundamental model of the world, *all* models, theories, formalizations, or simulations are thus limited simply because they are finite artifacts.

Gödel incompleteness does not imply (as Gödel himself believed) a pre-existing Platonic realm, any more than the ability to construct an infinite set implies its pre-existence. Nor does it imply that human intelligence is categorically superior to machine intelligence. If it is superior at present, this is because computers are not yet agents or mathematicians in their own right. The superiority of the human mind lies in its ability to adaptively reconfigure itself indefinitely, redefining ever more inclusive formal systems and points of view. The assumption that machines could never have this ability is mere prejudice, based on an outdated concept of the machine as a fixed artifact created from top down by a mind external to it—a static product of human definition, a tool rather than a tool user. On the other hand, an autonomous machine with better than human general intelligence would necessarily elude human definition and control.

5. Discrete/continuous

The natural numbers abstract the integrity of discrete "objects," such as human beings perceive in their physical or mental environment, and which they themselves exemplify as individuals. The finite steps of a proof or verification exemplify discrete acts of an agent upon a world of such objects. This corresponds to primate experience and action in an environment consisting of countable things, whether tangible or abstract. Groupings of such objects are abstracted as sets. Definability is the power to specify the nature of the element or set. Decidability is the power to determine whether an element belongs in a set. Computability is the power to generate the set with a succinct rule. Clearly these "abilities" involve agents and are not exclusively properties of the objects involved. While the non-computable reals cannot even be specified, that sort of obstacle has never stymied mathematicians. Progress involves defining objects (even paradoxical ones such as the square root of minus one or different orders of infinity) and manipulations upon them; and computation executes these manipulations. Nothing inherent in either mathematics or human nature prevents new mathematical actions that could treat non-computable numbers as objects of manipulation—though the utility of doing so is a separate question.

One must beware the assumption that an alternative mathematics could only be wrong, or not math at all.[ii] What if, at some level, the world does *not* consist of discrete objects and actions? Would another concept of mathematics and of computation better describe that level, one not based on natural numbers or discrete steps of reasoning? Could there be such a thing as analog proof, which does not rely on a linear sequence of logical steps, each critically dependent on the previous one? Non-locality and the unorthodox behavior of microphysical entities suggests the possibility of such a level of physical reality. For, like logic itself, the concepts of classical physics are an effect of scale: the physical size of human observers and the things with which they have ordinary dealings is of a vastly different order than the size of micro entities. All macroscopic measurement is analog insofar as it does not merely answer a yes/no question by detecting a presence or absence. In contrast, microscopic measurements tend to be digital-seeming yes/no detection events, whose collective pattern must be compressed as a law, with the precision that comes of large numbers or repetitions rather than the sort of precision of definition that characterizes classical physics. Yet, even an analog representation is a compression, for the only perfect analog of the universe is the universe itself.

Imagine another sort of creature, itself amorphous and living in a continuous medium without discernable objects.[iii] What kind of mathematics might it conceive, if at all? (Indeed, what kind of mathematics might be appropriate for plants as agents, as opposed to primates?) Alternatively, imagine a hypothetical observer the size of a proton. What would constitute objects in its environment and what kind of math would it need? A hypothetical amorphous creature on *our* scale might perceive and adjust to changes in its environment continuously in time, without the need to deal with the threats and opportunities posed by definite objects and discernable events. An amorphous or non-localized creature might have means to change its (non-discrete) environment—for example, through chemical emissions. (Indeed, this is how most selfregulation works *within* the organism.) Such an entity would hardly be distinct from its environment in the way that bounded organisms are. “Decision” would be pointless in the absence of discrete events. “Computation” would not be digital but direct and appropriate co-variant response to change.[iv] What need or use could such an entity have for a digitally-based model or theory of the world—or for “laws” as algorithmic compressions of its input? The above creature would need to be able to respond in real time fast enough to deal with changes co-variantly, as though apparent objects were continuously varying fields. Conversely, the very appearance of an object cognitively signifies the inability (or needlessness) to adapt quickly enough to continuous micro changes—requiring (or permitting) a higher-level strategy. The human world, of course, appears to consist of objects; though our science can reduce these to continuous fields, our cognition does not. Of course, our distance senses may obviate the need for immediate response, allowing our science to pose questions unrelated to it. One question worth considering is: how might a mathematics that is not based on discrete objects and events make the world on the smallest and largest scales more comprehensible to us?

Uncertainty relations are not specific to the quantum scale, but represent a general property of a class of defined systems. There is a similar trade-off on the macroscopic scale in the information needed to establish position versus momentum, as between other “conjugate variables” where separate measurements are required to determine their values. Perhaps a *physical* interpretation of Heisenberg’s relations appeals because we do not fancy the idea that the objects involved (e.g. particles with position and momentum) might behave in a way inconsistent with what we expect of them simply as objects. Yet quantum objects *do* defy our intuitive expectations, according to the logic derived from experience with classical objects. This is not an inconsistency in the physical world but in our human worldview. In fact, the intuitive *logic of objects* is profoundly inconsistent to begin with! For, on the one hand, an object is integral, an indivisible whole, an individual. On the other hand, an object is extended in space; a process is extended in time. Extended things can consist of parts or subdivisions conceptually, regardless of whether they can actually be so divided physically. (Hence the notion of the continuum, either mathematical or physical.) The mathematical counterpart of this inconsistency entrains the problems of infinities and infinitesimals that have occupied mathematicians for centuries since Zeno. While intuitions about infinity and infinite divisibility extrapolate experience on the human scale, there is no *a priori* reason to assume they hold in unfamiliar domains. We are tempted to regard some particles as truly elementary, if only because we do not have the energy resources to split them into something more fundamental. But perhaps one also simply balks at the idea of unending complexity all the way down, not to mention unending infinity all the way up. Nevertheless, there could well be a purely physical reason for an ultimate bottom to physical reality, or for a finite universe, whether or not we can know it or deem it logical.

Furthermore, we tend to attribute the nature of the whole to the nature of its parts (reductionism). What happens when a thing has no internal parts, as in the case of an indivisible object (Euclid’s definition of a point)? Its behavior and even identity must refer instead to other objects or points comprising a more inclusive whole. If electrons, for example, are truly indivisible, there is nothing by which to distinguish them but position or state, which remain for us uncertain.[v]

On the other hand, anything problematic or paradoxical can simply be embraced as axiomatic, just as imaginary numbers were. For example, the idea of “perfectly hard” elastic solids implies instantaneous transmission of forces, which is contrary to general experience and defies continuity. In physics, one overcomes this difficulty with the concept of *field*, which accommodates continuous action through time. But the field concept was derived from physical experience of media potentially composed of parts, so there remains a tacit internal structure. One might wonder why force should propagate in a continuum from one infinitesimal point to another at all! On the other hand, one can simply posit such transmission axiomatically—as when electrons and virtual photons are held to account for the transfer of momentum through some "exchange"—without imagining the details of a mechanical process or being concerned whether particles even take up space or causal processes take up time. A general problem is then: How to distinguish the "fundamental" particle's intrinsic integrity (if such exists) from its discreteness as a product of definition, as a theoretical construct whose properties are simply posited? Such problems are mostly skirted in the conceptual development of physics, allowing business as usual. And neither have they barred progress in mathematics. On the other hand, we may pay an ultimate price for what we believe to be progress, having gotten out onto theoretical limbs from which there is no retracing steps without finally resolving such questions.

6. Computability

It is currently fashionable to imagine that the physical universe is no more than a subset of mathematics or even a vast computer. However, mathematics reflects our human experience of the world and is in no wise the cause of it. If the unfolding of the real world in time were nothing but the logical development of a deductive system—in other words, if the world were truly a deterministic system—randomness would simply be a function of ignorance. While the *mathematical* equivalent of randomness is non-computability, which exists through the capacity of mind to transcend the formalisms it creates, it would be foolish to imagine that randomness in nature reflects no more than a mental state.

It might seem puzzling that the non-computable reals could arise within a logical system premised on computability; but this inconsistency lies within our thinking rather than within a pre-existing mathematical reality. Similarly, one might wonder why some irrational numbers (such as π) are computable while others are not. But the question is loaded. The standard answer is that an arbitrary string of decimal digits (what most irrational numbers are supposed to be) displays no pattern that could be expressed by an algorithm. However, this puts the cart before the horse, for π was not revealed to be computable by identifying a pattern within its pre-existing decimal expansion. On the contrary, its definition as a unique geometric ratio already suggested methods for generating that expansion. Numbers such as π and e stand out as relations significant for human purposes, while arbitrary sequences do not. The question seems innocuous only because, post hoc, irrational numbers were defined as arbitrary non-terminating decimals.

7. Conclusion

Non-computability arises in the context of a quest to seek perfect accuracy using tools that can only approximate (as in the limit of a series). It is a by-product of Gödellian self-transcendence. In contrast, unpredictability (randomness) is a function not only of the subject's limited knowledge, but also—crucially—of the fundamentally inscrutable nature of the world itself, which inevitably eludes any final or complete account. It is this very elusiveness that signifies the world as *real*, independent of human thought, and distinct from the mathematical simulation. A digital physics, (like the metaphysical notion that the universe is a digital computer) would guarantee computability; but it would not be true to the reality of nature. Self-contained in the way that all formalisms are, it would regress to the scholasticism that science was conceived to avoid. While practical, ultimately it would not satisfy even the mathematical mind, which is ordained to transcend its own creations.

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[i] Cf. Yampolskiy's idea of 'proof verifier' as the mathematical version of an epistemic subject or observer in physics [Roman V. Yampolskiy "Unverifiability, Unexplainability & Unpredictability" https://fqxi.org/community/forum/topic/3380]. However, verification is a different function than cognition.

[ii] In a broader other context, I refer to the revisionist hindsight that interprets older mathematical or physical ideas as wrong or incomplete in the terms of current ideas, as an example of "the problem of cognitive domains." See *https://philpapers.org/rec/BRUTPO-29*

[iii] Cf. Fred Hoyle's 1957 sci-fi novel *The Black Cloud*

[iv] This is the trivial sense in which some view the physical universe as a computer.

[v] Peter Jackson asks, "is anything in the universe precisely identical?" [Jackson "Blondes, Brunettes, and the Flaw of Excluded Middle" https://fqxi.org/community/forum/category/31427]. This must be approached as a genuine, not rhetorical question.

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