RANDOMNESS, COMPUTABILITY, AND EMBODIMENT

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Abstract

Physical and mathematical concepts ultimately reflect the nature of the embodied human organism as well as its world. Non-computability in mathematics and randomness in physics refer to limits within an epistemic relationship between subject and object. Non-computability signifies the ability of a self-reflective agent to transcend its own formulations; in contrast, randomness in nature signifies the natural world's ability to transcend formalization in thought.

1. Subject-object

What does the physically embodied nature of the human epistemic subject imply for the properties and uses of mathematics? The traditional answer is: nothing. In whole or in part, however, mathematics will be viewed here as a *simulation* created by the human organism for adaptive purposes, just as ordinary perception is a virtual reality that simulates the world outside the skull. The purpose of the mathematics simulation is to characterize the most general properties of the physical world in an empoweringly abstract and compressed way, especially to facilitate prediction—a view early propounded by J. S. Mill. The expressive possibilities within the simulation are set by how it is configured. Like any simulation, mathematics constitutes a "toy" world that mirrors reality yet potentially diverges from it.

An epistemic subject is an embodied agent whose natural focus is its environment. If this subject is aware of its agency, it is capable of self-reference and has a dual focus: the external world, and also its own experience and thought, which may include linguistic and other conceptual objects in a distinctly mental realm. Distinguishing these realms can be problematic. Science is a strategy to disentangle the subjective from the objective aspects of experience, first by restricting its language to external objects, whether tangible or abstract, thereby avoiding self-reference (hence, 'electron' is a scientific object while 'concept of electron' is not); then, by further eliminating idiosyncratic elements, imposing strict protocols, and standardizing interchangeable observers.[1] What escapes this net, however, is what standardized observers have in common that is not on the table for discussion—for example, properties belonging commonly to all observers at the species level. Mathematics traditionally presumes that its elements are absolute in a way that the contingencies represented in science are not—in other words, that such commonalities have already been eliminated or are irrelevant. As a form of cognition, mathematics confidently substitutes its simulation to represent collective cumulative experience on the most general level. The simulation conforms to reason because reason itself has been abstracted from the most

[1] Cf. Yampolskiy's idea of 'proof verifier' as the mathematical version of an epistemic subject or observer in science.

generic and indisputable aspects of the world as seen by the human organism. Yet, that is not sufficient claim to the a priori status of absolute truths that would necessarily be perceived by any possible subject.

The world is apparently subdivided into distinct objects, with interrelationships that implicitly involve the observer (forming a literal triangle in the case of two objects observed in space). The observer is motivated to categorize these relationships in various ways (cause/effect, closer/farther, uniformly or non-uniformly changing, etc.) The notion of mathematical *function* expresses, in a precise but abstract way, how one object of thought relates to another, especially over time, since change in the real world is of vital significance for the observer as an embodied organism. Where possible, mathematics goes helpfully beyond noting qualitative relationships, to express exactly how one "variable" changes continuously with another. To put it differently, the idea of *variable* is itself a mathematical abstraction. The trade-off for this gain is that *only* those patterns or relationships are considered which can be so formulated. (Thus, velocity is precisely expressed as uniform change of location; acceleration as uniform change of velocity; change of acceleration as a further differential.) But these are ideals of analysis to which most real phenomena may not necessarily conform. Special relationships are identified, as physically significant, which can be expressed with such mathematical tools as differential equations; but the complete reality of any given phenomenon—and certainly the world as a whole—may not correspond in detail to any collection of such expressions. How should Brownian movement, for example, be expressed?

It may be claimed that these relationships exist "objectively," rather than as an observer's assertions. This reifying tendency itself is a consequence of the natural focus of embodied subjects upon the environment that holds over them the power of life and death. Thus, limits of scientific knowledge center on predictability, but as though predictability were an objective property of the externals concerned rather than reflecting the observer's needs, limits, and state of knowledge. For example, the concept of *information* in physics tends to be objectified as a quantifiable property of systems themselves, even as a fundamental physical entity like energy. However, what we take to be information concerns structure, the perception of which is relative to human needs and goals. Indeed, any subject's actions and perceptions can at most be *co-determined* by the world external to the epistemic agent. Ignoring the agent's role gives a skewed, if useful, view of the world.

Concerns about the power and limits of mathematics have ultimately to do with prediction in the real world. This gives a certain urgency to the consistency of the mathematics simulation, without which its usefulness is compromised. Apart from their intrinsic fascination, we care about potential discoveries in the mathematics simulation itself because they might affect how we fare in reality. If the self-consistency of mathematics reflects nature's intrinsic self-consistency, then it is unsurprising to find it consistent with parts of natural reality newly discovered or outside familiar domains. The question is then why *nature* is self- consistent, if indeed it is.

One is naturally inclined to view the world in idealized ways that lend themselves to effectively deciding the truth of propositions and predicting the course of events. Hence, the law of excluded middle and the historic focus on systems describable with simple linear equations, manually solvable. Idealizing these predictive capacities resulted in the Laplacian faith that anything past or present could be calculated, given a precise enough input. The subsequent realization that non-linear processes prevail in the real world followed upon Poincaré's study of the 3-body problem and the advent of digital computers, through which 'deterministic chaos' was rediscovered by Lorenz. This computational finding had the real-world consequence that inputs could not be measured accurately enough to allow effective prediction of many phenomena humanly important. Precise inputs can be *specified*, but the outputs (though reliably deduced) might not correspond to empirical reality. In other words, the world is *not* deterministic in the way Laplace's generation had hoped. There was never any reason to suppose that it is, save the wishful thought that the world should be commensurate with reason, the future predictable.

2. Map/territory

The determinism that was once supposed to be an inherent property of *nature* is instead a defining property of *equations* and other conceptual systems, by virtue of their being *products of definition*. The map is deterministic, not the territory. Determinism is a property of the mathematical simulation, not of physical reality in either the classical or quantum realm. In other words, determinism is nothing more than logical implication, and causal processes in the natural world are not a matter of logical implication as they are in mathematics. Moreover, being empirical, all scientific knowledge is essentially statistical. The difference between the classical and quantum domains is the brand of statistics, which depends on whether the observer can keep track of individual particles (objects).

The "unreasonable effectiveness" of the mathematics simulation for describing the world is but a special instance of the effectiveness of cognition in general to relate the domain of nature to our conceptual maps of it. Behind the utility of prediction tacitly lies the human project to master nature and create a specifically human environment. This artificial environment is conceptual as well as, and often before, it is physical. The conceptual map bears a symbolic relation to the territory that works for specific purposes. If the map seems to be a likeness or copy, this illusion is the result of a strange loop whereby a self-aware agent can conceive the territory only in the terms of the map. The familiar cognitive map we call phenomenal consciousness, which we take for reality on a daily basis, must be distinguished from the inaccessible domain it maps—which Kant called the *noumenon*.

The mapmaker (the human brain) is metaphorically in the position of a pilot flying by instrument or a submarine navigator charting the underwater world. Like these characters in their capsules, the brain is sealed within the skull, with access to the outside only through electro-chemical signals from remote sensing instruments, of which it must learn to make sense. *Unlike* these characters, it has never had any other access to the external world! (There are no windows or doors in the aircraft or submarine.) This constrains the nature of the map in a unique way: it is not a literal image but no more than a device that facilitates successful navigation—where success means simply that the vessel is not destroyed. The navigator or pilot does not survive because the map is true (which puts the cart before the horse). Rather the map is "true" because the navigator or pilot survives!

3. Decision

Decisions are made by agents with respect to needs. Although never forthcoming, perfect information is desirable for action. The amount of information a message contains is related to the number of yes/no decisions required to characterize it unequivocally. If the agent cannot decide some questions, the information remains ambiguous, its quantity indefinite. An artifact (such as a message, text, theory, or model) is potentially characterized by precisely definite information; but there is no guarantee that the reality to which it refers can be so characterized. (Indeed, there is little reason to suppose that the world is a message or other artifact!) It is the agent's prerogative (and perhaps mandate) to *force* decisiveness where a justification has not been legitimized; but this merely reflects the biological and psychological need for certainty.

Perfect certainty pertains only to deductive truths—that is, things *defined* to be so in the first place (axioms) and things (theorems) that can be deduced from them according to rules that are universally accepted. Mathematical or logical propositions (being clearly defined) have the advantage that the community of mathematicians is at least marginally more likely to agree that a theorem has been proven than the scientific community is likely to agree that a theory is true. The nature of proof in the two instances is different, even though both are intersubjective and debatable. In contrast to the scholasticism it superseded, the whole idea behind the Scientific Revolution is that theories of nature should be demonstrated empirically rather than deductively.

Scientific consensus nowadays depends on conventions for evaluating statistical error—as in deciding, for example, whether a high energy experiment confirms the existence of a particle. Nevertheless, deduction is still a perennial ideal in modern science. The hope for a theory of everything, for example, rests on the notion that all physical properties, even basic physical constants, should ultimately be derived from a few first principles. Many physical laws were in fact logically deduced before being confirmed empirically. Yet, such deductions are only possible because they deal with universal properties of things, whose more specific and seemingly contingent properties (such as weight) cannot contradict them. Hence, Galileo's conclusion that the weight of objects does not affect their rate of fall came to him deductively in a thought experiment, not by dropping objects from the Tower of Pisa. By supposing the contrary, it was clear that objects could not fall at different rates without contradicting the very fact of being separate objects! Similarly, it is clear that forces cannot be instantaneously transmitted (action at a distance) without violating common-sense notions involving space and time.

It might seem surprising that the physical world should appear internally consistent and logically organized ("rational") in the very way that math is. But this is no coincidence, because math was modelled on the world and not the other way around. Moreover, in truth, the parallel between math and nature is only partial. The very strength of science is to reveal the apparent *in*consistencies in nature—for example, the "illogical" behavior of quantum objects. That discrepancy stems from the fact that our habitual logic itself generalizes the behavior of things in the familiar macroscopic realm and then proceeds to institutionalize these generalizations in its "laws." The notion of *a priori* truth, gleaned from such observed behavior, is not absolute but rather characterizes those levels of cognition that are common, if not universal for the species, and thus come to seem not contingent but necessary.

4. Objects

Mathematics deals with grand generalities concerning discernable "things." Computability theory is thus concerned mainly with the natural numbers, which abstract the integral properties of "objectness." However, these generalities have been formalized as the rules, operations, and entities of a deductive system (namely, arithmetic). Such a system is a free-standing product of definition categorically distinct from the world itself. The truths of logic and of deductive systems are true by definition. This is so whether or not they have even yet been discovered or been formally derived from currently known truths. They are true whether or not they ever represent known physical quantities. Mathematical truth and physical truth are distinct domains, which overlap to an indefinite degree. As in the real world, we assume that some relationships in the mathematical simulation remain to be found, however complete the existing formalization. While this assumption is intuitive and counter to the ideal of perfect knowledge, it was formally proven by Gödel. It was further established (for example, by Chaitin) that there are infinitely more non-computable numbers than computable ones in the mathematics simulation, paralleling the preponderance of "deterministically" chaotic processes in nature.

It may be useful to distinguish between those relationships within the natural world that must follow from its general properties (as formalized in logic and mathematics) from those that cannot be so deduced. The notion of *computability* may afford this distinction. Non-computability presents a limit on the ability of the mathematics simulation (or any other) to capture the real world. While math is the most general and fundamental model of the world, *all* models, theories, formalizations, or simulations are thus limited simply because they are finite artifacts.

Gödel incompleteness does not imply (as Gödel himself believed) a pre-existing Platonic realm, any more than the ability to construct an infinite set implies its pre-existence. Nor does it imply that human intelligence is categorically superior to machine intelligence. If it still is superior at present, this is because computers are not yet agents or mathematicians in their own right. The superiority of the human mind lies in its ability to adaptively reconfigure itself indefinitely, redefining ever more inclusive formal systems and points of view. The assumption that machines could never have this ability is mere prejudice, based on an outdated concept of the machine as a fixed artifact created from top down by a mind external to it—a static product of human definition, a tool rather than a tool user. On the other hand, an autonomous machine with better than human general intelligence would necessarily elude human definition and control.

5. Discrete/continuous

The natural numbers abstract the integrity of discrete "objects," such as human beings perceive in their physical or mental environment, and which they themselves exemplify as individuals. The finite steps of a proof or verification exemplify discrete acts of an agent upon a world of such objects. This corresponds to primate experience and action in an environment consisting of countable things, whether tangible or abstract. Groupings of such objects are abstracted as sets. Definability is the power to specify the nature of the element or set. Decidability is the power to determine whether an element belongs in a set. Computability is the power to generate the set with a succinct rule. Clearly these "abilities" involve agents and are not exclusively properties of the objects involved. While the non-computable reals cannot even be specified, that sort of obstacle has never stymied mathematicians. Progress involves defining new objects (even paradoxical ones such as the square root of minus one or different orders of infinity) and manipulations upon them. Computation then executes these manipulations. Nothing inherent in either mathematics or human nature prevents new mathematical actions that could treat non-computable numbers as objects of manipulation—though the utility of doing so is a separate question.

Could there be an alternative mathematics? What if, at some level, the world does not consist of discrete objects and actions? Would another concept of mathematics and of computation better describe that level, one not based on natural numbers or discrete steps of reasoning? Non-locality and the unorthodox behavior of microphysical entities suggests the possibility of such a level of physical reality. The concepts of classical physics are a function of scale: the physical size of human observers, and the things with which they have ordinary dealings, is of a vastly different order than the size of micro entities. Macroscopic measurement is relatively analog, insofar as it does not merely answer a yes/no question by detecting a presence or absence. In contrast, microscopic measurements are often digital-seeming detection events, whose collective pattern must be compressed as an analog-seeming law, with the sort of precision that comes of large numbers or repetitions.

Imagine another sort of creature, itself amorphous[2] and living in a continuous medium without discernable objects. What kind of mathematics might it conceive, if any? (Indeed, what kind of mathematics might a plant conceive, as distinguished from a primate?) Alternatively, imagine a hypothetical observer the size of a proton. What would constitute objects in its environment and what kind of math would it need? A hypothetical creature on *our* scale, living in an amorphous environment, might perceive and adjust to environmental changes continuously in time, without the need to deal with the threats and opportunities posed by definite objects and discernable events. It might respond continuously through chemical emissions, for example. (Indeed, this is how most self-regulation works *within* organisms.) “Decision” would be pointless in the absence of discrete events. “Computation” would not be digital but direct and appropriate co-variant response to change—in other words, analog. What need or use could such an entity have for a digitally-based model or theory of the world—or for “laws” as algorithmic compressions of a digital input?

Such a creature would need to be able to respond in real time fast enough to deal with changes effectively. Since discreteness is not an absolute property, but relative to scale and to the observer, this creature would hypothetically be able to deal with apparent objects as though they were continuously varying fields. Conversely, the very appearance of an object cognitively signifies the inability (or needlessness) of a creature to adapt quickly enough to continuous micro changes—in compensation for which it invokes higher-level strategies. The human world, of course, appears to consist of objects. Though our science can reduce these to continuous fields, our cognition does not. Our distance senses obviate the need for immediate response to distant fields perceived as objects of interest, so that continuous adjustment is not required. A question

[2] For example, as in Fred Hoyle’s 1957 sci-fi novel *The Black Cloud.*

worth considering is: how might a mathematics that is not based on discrete objects and events make the world on the smallest and largest scales more comprehensible to us?

Uncertainty relations, such as Heisenberg's, are not specific to the quantum scale, but represent a general property of a class of defined systems. There is a similar trade-off on the macroscopic scale in the information needed to establish position versus momentum, as between other "conjugate variables" where separate measurements are required to determine their values. Quantum objects defy our intuitive expectations, based on experience with classical objects. This does not reflect an inconsistency in the physical world but in our human worldview. In fact, the intuitive *logic of objects* is profoundly inconsistent to begin with. For, on the one hand, an object is integral, an indivisible whole, an individual. On the other hand, it is extended in space, and a process is extended in time. Conceptually, extended things consist potentially of parts or subdivisions, regardless of whether they can actually be so divided physically. The mathematical counterpart of this problem is inherent in the notion of the continuum, entraining the problems of infinities and infinitesimals that have occupied mathematicians for centuries since Zeno. While intuitions about infinity and infinite divisibility extrapolate experience on the human scale, there is no *a priori* reason to assume they hold in unfamiliar domains. If we are tempted to regard some particles as truly elementary, it may only be because we do not have the energy resources to split them into something more fundamental. But perhaps one also simply balks at the idea of unending complexity all the way down, not to mention unending infinity all the way up. Nevertheless, there could well be a purely physical reason for an ultimate bottom to physical reality, or for a finite universe, whether or not we think it makes intuitive sense.

Reductionism is the tendency to attribute the nature of the whole to the nature of its parts. What happens when a thing *has* no internal parts, as in the case of an allegedly indivisible object or one not extended in space (Euclid's definition of a point)? Its behavior and even identity must refer instead to other objects or points comprising a more inclusive whole. If electrons, for example, are truly indivisible, there is nothing by which to distinguish them but position or momentum, which remain for us uncertain. Moreover, they may be discrete by definition, as units of charge rather than as objects.

On the other hand, anything problematic or paradoxical can simply be embraced as axiomatic, just as imaginary numbers were. For example, the idea of "perfectly hard" elastic solids implies instantaneous transmission of force, which is contrary to general experience and defies continuity in time. In physics, one overcomes this difficulty with the concept of *field*, which accommodates continuous action through time. But the field concept was derived from physical experience of media potentially composed of parts, so there remains a tacit internal structure. One might wonder why force should propagate in a continuum from one infinitesimal point to another at all! On the other hand, one can simply posit such transmission axiomatically—as when electrons and virtual photons are held to account for the transfer of momentum through some "exchange"—without imagining the mechanical details of the process, or being concerned whether particles even take up space or whether causal processes take up time. A general problem is then: How to distinguish the "fundamental" particle's intrinsic integrity (if such exists) from its discreteness as a product of definition, as a theoretical construct whose properties are simply posited? Such impediments are mostly skirted in the conceptual development of physics, allowing business to carry on as usual. Neither have they barred progress in mathematics. On the other hand, we may pay an ultimate price for what we believe to be progress, having swept fundamental inconsistencies under the rug. We may have gotten out onto theoretical limbs from which there is no retracing steps without finally resolving such questions.

6. Computability

It is currently fashionable to imagine that the physical universe is no more than a subset of mathematics or even a vast computer. However, mathematics reflects our human experience of the world and is in no wise the cause of it. If the unfolding of the real world in time were nothing but the logical development of a deductive system—in other words, if the world were truly a deterministic system—randomness would be purely a function of ignorance. The *mathematical* equivalent of randomness is non-computability, which exists through the capacity of mind to transcend the formalisms it creates (Gödellian self-transcendence). It would be foolish to imagine that randomness in nature reflects no more than a mental state.

It might seem puzzling that the non-computable reals could arise within a logical system premised on computability; but this inconsistency lies within our thinking rather than within a pre-existing mathematical reality. Similarly, one might wonder why some irrational numbers (such as π) are computable while others are not. But the question is loaded. The standard answer is that an arbitrary string of decimal digits (what most irrational numbers are supposed to be) displays no pattern that could be expressed by an algorithm. However, this puts the cart before the horse, for π is no arbitrary string. It was not revealed to be computable by identifying a pattern within its decimal expansion. On the contrary, its definition as a unique geometric ratio already suggested methods for generating that expansion. Numbers such as π and e stand out as relations significant for human purposes, while "arbitrary" sequences do not.

7. Conclusion

Non-computability arises in the context of a quest to seek perfect accuracy using tools that can only approximate (as in the limit of a series). In contrast, unpredictability (randomness) is a function not only of the subject's limited knowledge, but also—crucially—of the fundamentally inscrutable nature of the world itself, which inevitably eludes any final or complete account. It is this very elusiveness that signifies the world as *real*, independent of human thought, and distinct from the mathematical simulation. A digital physics, (like the metaphysical notion that the universe is a digital computer) would guarantee computability; but it would not be true to the reality of nature. Self-contained in the way that all formalisms are, it would constitute a regression to the scholasticism that science was conceived to avoid. While practical, ultimately it would not satisfy even the mathematical mind, which is ordained to transcend its own creations.

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